Übung 1.
Berechne einige der Legendre-Symbole

(a) $\left(\frac{-213}{1931}\right)$	(b) $\left(\frac{86}{1217}\right)$	(c) $\left(\frac{831}{877}\right)$	(d) $\left(\frac{1248}{1423}\right)$	(e) $\left(\frac{21}{31}\right)$
(f) $\left(\frac{327}{757}\right)$	(g) $\left(\frac{478}{1249}\right)$	(h) $\left(\frac{61}{541}\right)$	(i) $\left(\frac{17}{43}\right)$	(j) $\left(\frac{-836}{1327}\right)$
(k) $\left(\frac{28}{241}\right)$	(l) $\left(\frac{-72}{587}\right)$	(m) $\left(\frac{-106}{239}\right)$	(n) $\left(\frac{221}{397}\right)$	(o) $\left(\frac{361}{1627}\right)$
(p) $\left(\frac{-35}{167}\right)$	(q) $\left(\frac{913}{1483}\right)$	(r) $\left(\frac{1315}{1567}\right)$	(s) $\left(\frac{-38}{997}\right)$	(t) $\left(\frac{220}{353}\right)$

Lösung 1.
(a)

$$\begin{aligned}\left(\frac{-213}{1931}\right) &= \left(\frac{-1}{1931}\right)\cdot\left(\frac{213}{1931}\right)\\ &= (-1)^{\frac{1931-1}{2}}\cdot\left(\frac{3}{1931}\right)\cdot\left(\frac{71}{1931}\right)\\ &= (-1)\cdot\left(\frac{2}{3}\right)\cdot(-1)^{\frac{3-1}{2}\cdot\frac{1931-1}{2}}\cdot\left(\frac{14}{71}\right)\cdot(-1)^{\frac{71-1}{2}\cdot\frac{1931-1}{2}}\\ &= -(-1)^{\frac{3-1}{2}}\cdot(-1)\cdot\left(\frac{2}{71}\right)\cdot\left(\frac{7}{71}\right)\cdot(-1)\\ &= -(-1)\cdot(-1)^{\frac{71^2-1}{8}}\cdot\left(\frac{1}{7}\right)\cdot(-1)^{\frac{7-1}{2}\cdot\frac{71-1}{2}}\\ &= -1\end{aligned}$$

(b)

$$\begin{aligned}\left(\frac{86}{1217}\right) &= \left(\frac{2}{1217}\right)\cdot\left(\frac{43}{1217}\right)\\ &= (-1)^{\frac{1217^2-1}{8}}\cdot\left(\frac{13}{43}\right)\cdot(-1)^{\frac{43-1}{2}\cdot\frac{1217-1}{2}}\\ &= 1\cdot\left(\frac{4}{13}\right)\cdot(-1)^{\frac{13-1}{2}\cdot\frac{43-1}{2}}\cdot 1\\ &= \left(\frac{2}{13}\right)^2\cdot 1\\ &= 1\end{aligned}$$

(c)

$$\begin{aligned}\left(\frac{831}{877}\right) &= \left(\frac{3}{877}\right)\cdot\left(\frac{277}{877}\right)\\ &= \left(\frac{1}{3}\right)\cdot(-1)^{\frac{3-1}{2}\cdot\frac{877-1}{2}}\cdot\left(\frac{46}{277}\right)\cdot(-1)^{\frac{277-1}{2}\cdot\frac{877-1}{2}}\\ &= 1\cdot 1\cdot\left(\frac{2}{277}\right)\cdot\left(\frac{23}{277}\right)\cdot 1\\ &= (-1)^{\frac{277^2-1}{8}}\cdot\left(\frac{1}{23}\right)\cdot(-1)^{\frac{23-1}{2}\cdot\frac{277-1}{2}}\\ &= -1\end{aligned}$$

(d)

$$\begin{aligned}\left(\frac{1248}{1423}\right) &= \left(\frac{2}{1423}\right)^5 \cdot \left(\frac{3}{1423}\right) \cdot \left(\frac{13}{1423}\right) \\ &= \left(\frac{2}{1423}\right) \cdot \left(\frac{1}{3}\right) \cdot (-1)^{\frac{3-1}{2}\cdot\frac{1423-1}{2}} \cdot \left(\frac{6}{13}\right) \cdot (-1)^{\frac{13-1}{2}\cdot\frac{1423-1}{2}} \\ &= (-1)^{\frac{1423^2-1}{8}} \cdot 1 \cdot (-1) \cdot \left(\frac{2}{13}\right) \cdot \left(\frac{3}{13}\right) \cdot 1 \\ &= -1 \cdot (-1)^{\frac{13^2-1}{8}} \cdot \left(\frac{1}{3}\right) \cdot (-1)^{\frac{3-1}{2}\cdot\frac{13-1}{2}} \\ &= 1\end{aligned}$$

(e)

$$\begin{aligned}\left(\frac{21}{31}\right) &= \left(\frac{3}{31}\right) \cdot \left(\frac{7}{31}\right) \\ &= \left(\frac{1}{3}\right) \cdot (-1)^{\frac{3-1}{2}\cdot\frac{31-1}{2}} \cdot \left(\frac{3}{7}\right) \cdot (-1)^{\frac{7-1}{2}\cdot\frac{31-1}{2}} \\ &= 1 \cdot (-1) \cdot \left(\frac{1}{3}\right) \cdot (-1)^{\frac{3-1}{2}\cdot\frac{7-1}{2}} \cdot (-1) \\ &= -1\end{aligned}$$

(f)

$$\begin{aligned}\left(\frac{327}{757}\right) &= \left(\frac{3}{757}\right) \cdot \left(\frac{109}{757}\right) \\ &= \left(\frac{1}{3}\right) \cdot (-1)^{\frac{3-1}{2}\cdot\frac{757-1}{2}} \cdot \left(\frac{103}{109}\right) \cdot (-1)^{\frac{109-1}{2}\cdot\frac{757-1}{2}} \\ &= 1 \cdot 1 \cdot \left(\frac{6}{103}\right) \cdot (-1)^{\frac{103-1}{2}\cdot\frac{109-1}{2}} \cdot 1 \\ &= \left(\frac{2}{103}\right) \cdot \left(\frac{3}{103}\right) \cdot 1 \\ &= (-1)^{\frac{103^2-1}{8}} \cdot \left(\frac{1}{3}\right) \cdot (-1)^{\frac{3-1}{2}\cdot\frac{103-1}{2}} \\ &= -1\end{aligned}$$

(g)

$$\begin{aligned}\left(\frac{478}{1249}\right) &= \left(\frac{2}{1249}\right)\cdot\left(\frac{239}{1249}\right)\\ &= (-1)^{\frac{1249^2-1}{8}}\cdot\left(\frac{54}{239}\right)\cdot(-1)^{\frac{239-1}{2}\cdot\frac{1249-1}{2}}\\ &= 1\cdot\left(\frac{2}{239}\right)\cdot\left(\frac{3}{239}\right)^3\cdot 1\\ &= (-1)^{\frac{239^2-1}{8}}\cdot\left(\frac{3}{239}\right)\\ &= 1\cdot\left(\frac{2}{3}\right)\cdot(-1)^{\frac{3-1}{2}\cdot\frac{239-1}{2}}\\ &= (-1)^{\frac{3-1}{2}}\cdot(-1)\\ &= 1\end{aligned}$$

(h)

$$\begin{aligned}\left(\frac{61}{541}\right) &= \left(\frac{53}{61}\right)\cdot(-1)^{\frac{61-1}{2}\cdot\frac{541-1}{2}}\\ &= \left(\frac{8}{53}\right)\cdot(-1)^{\frac{53-1}{2}\cdot\frac{61-1}{2}}\cdot 1\\ &= \left(\frac{2}{53}\right)^3\cdot 1\\ &= \left(\frac{2}{53}\right)\\ &= (-1)^{\frac{53^2-1}{8}}\\ &= -1\end{aligned}$$

(i)

$$\begin{aligned}\left(\frac{17}{43}\right) &= \left(\frac{9}{17}\right)\cdot(-1)^{\frac{17-1}{2}\cdot\frac{43-1}{2}}\\ &= \left(\frac{3}{17}\right)^2\cdot 1\\ &= 1\end{aligned}$$

(j)

$$\begin{aligned}
\left(\frac{-836}{1327}\right) &= \left(\frac{491}{1327}\right) \\
&= \left(\frac{345}{491}\right) \cdot (-1)^{\frac{491-1}{2} \cdot \frac{1327-1}{2}} \\
&= \left(\frac{3}{491}\right) \cdot \left(\frac{5}{491}\right) \cdot \left(\frac{23}{491}\right) \cdot (-1) \\
&= -\left(\frac{2}{3}\right) \cdot (-1)^{\frac{3-1}{2} \cdot \frac{491-1}{2}} \cdot \left(\frac{1}{5}\right) \cdot (-1)^{\frac{5-1}{2} \cdot \frac{491-1}{2}} \cdot \left(\frac{8}{23}\right) \cdot (-1)^{\frac{23-1}{2} \cdot \frac{491-1}{2}} \\
&= -(-1)^{\frac{3-1}{2}} \cdot (-1) \cdot 1 \cdot 1 \cdot \left(\frac{2}{23}\right)^3 \cdot (-1) \\
&= -(-1) \cdot \left(\frac{2}{23}\right) \\
&= (-1)^{\frac{23^2-1}{8}} \\
&= 1
\end{aligned}$$

(k)

$$\begin{aligned}
\left(\frac{28}{241}\right) &= \left(\frac{2}{241}\right)^2 \cdot \left(\frac{7}{241}\right) \\
&= 1 \cdot \left(\frac{3}{7}\right) \cdot (-1)^{\frac{7-1}{2} \cdot \frac{241-1}{2}} \\
&= \left(\frac{1}{3}\right) \cdot (-1)^{\frac{3-1}{2} \cdot \frac{7-1}{2}} \cdot 1 \\
&= -1
\end{aligned}$$

(l)

$$\begin{aligned}
\left(\frac{-72}{587}\right) &= \left(\frac{-1}{587}\right) \cdot \left(\frac{72}{587}\right) \\
&= (-1)^{\frac{587-1}{2}} \cdot \left(\frac{2}{587}\right)^3 \cdot \left(\frac{3}{587}\right)^2 \\
&= (-1) \cdot \left(\frac{2}{587}\right) \cdot 1 \\
&= -(-1)^{\frac{587^2-1}{8}} \\
&= 1
\end{aligned}$$

(m)

$$\begin{aligned}\left(\frac{-106}{239}\right) &= \left(\frac{-1}{239}\right)\cdot\left(\frac{106}{239}\right)\\ &= (-1)^{\frac{239-1}{2}}\cdot\left(\frac{2}{239}\right)\cdot\left(\frac{53}{239}\right)\\ &= (-1)\cdot(-1)^{\frac{239^2-1}{8}}\cdot\left(\frac{27}{53}\right)\cdot(-1)^{\frac{53-1}{2}\cdot\frac{239-1}{2}}\\ &= -1\cdot\left(\frac{3}{53}\right)^3\cdot 1\\ &= -\left(\frac{3}{53}\right)\\ &= -\left(\frac{2}{3}\right)\cdot(-1)^{\frac{3-1}{2}\cdot\frac{53-1}{2}}\\ &= -(-1)^{\frac{3-1}{2}}\cdot 1\\ &= 1\end{aligned}$$

(n)

$$\begin{aligned}\left(\frac{221}{397}\right) &= \left(\frac{13}{397}\right)\cdot\left(\frac{17}{397}\right)\\ &= \left(\frac{7}{13}\right)\cdot(-1)^{\frac{13-1}{2}\cdot\frac{397-1}{2}}\cdot\left(\frac{6}{17}\right)\cdot(-1)^{\frac{17-1}{2}\cdot\frac{397-1}{2}}\\ &= \left(\frac{6}{7}\right)\cdot(-1)^{\frac{7-1}{2}\cdot\frac{13-1}{2}}\cdot 1\cdot\left(\frac{2}{17}\right)\cdot\left(\frac{3}{17}\right)\cdot 1\\ &= (-1)^{\frac{7-1}{2}}\cdot 1\cdot(-1)^{\frac{17^2-1}{8}}\cdot\left(\frac{2}{3}\right)\cdot(-1)^{\frac{3-1}{2}\cdot\frac{17-1}{2}}\\ &= (-1)\cdot 1\cdot(-1)^{\frac{3-1}{2}}\cdot 1\\ &= 1\end{aligned}$$

(o)

$$\begin{aligned}\left(\frac{361}{1627}\right) &= \left(\frac{19}{1627}\right)^2\\ &= 1\end{aligned}$$

(p)

$$\begin{aligned}\left(\frac{-35}{167}\right) &= \left(\frac{-1}{167}\right)\cdot\left(\frac{35}{167}\right)\\ &= (-1)^{\frac{167-1}{2}}\cdot\left(\frac{5}{167}\right)\cdot\left(\frac{7}{167}\right)\\ &= (-1)\cdot\left(\frac{2}{5}\right)\cdot(-1)^{\frac{5-1}{2}\cdot\frac{167-1}{2}}\cdot\left(\frac{6}{7}\right)\cdot(-1)^{\frac{7-1}{2}\cdot\frac{167-1}{2}}\\ &= -(-1)^{\frac{5^2-1}{8}}\cdot 1\cdot(-1)^{\frac{7-1}{2}}\cdot(-1)\\ &= 1\end{aligned}$$

(q)

$$\begin{aligned}\left(\frac{913}{1483}\right) &= \left(\frac{11}{1483}\right)\cdot\left(\frac{83}{1483}\right)\\ &= \left(\frac{9}{11}\right)\cdot(-1)^{\frac{11-1}{2}\cdot\frac{1483-1}{2}}\cdot\left(\frac{72}{83}\right)\cdot(-1)^{\frac{83-1}{2}\cdot\frac{1483-1}{2}}\\ &= \left(\frac{3}{11}\right)^2\cdot(-1)\cdot\left(\frac{2}{83}\right)^3\cdot\left(\frac{3}{83}\right)^2\cdot(-1)\\ &= 1\cdot\left(\frac{2}{83}\right)\cdot 1\\ &= (-1)^{\frac{83^2-1}{8}}\\ &= -1\end{aligned}$$

(r)

$$\begin{aligned}\left(\frac{1315}{1567}\right) &= \left(\frac{5}{1567}\right)\cdot\left(\frac{263}{1567}\right)\\ &= \left(\frac{2}{5}\right)\cdot(-1)^{\frac{5-1}{2}\cdot\frac{1567-1}{2}}\cdot\left(\frac{252}{263}\right)\cdot(-1)^{\frac{263-1}{2}\cdot\frac{1567-1}{2}}\\ &= (-1)^{\frac{5^2-1}{8}}\cdot 1\cdot\left(\frac{2}{263}\right)^2\cdot\left(\frac{3}{263}\right)^2\cdot\left(\frac{7}{263}\right)\cdot(-1)\\ &= -(-1)\cdot 1\cdot 1\cdot\left(\frac{4}{7}\right)\cdot(-1)^{\frac{7-1}{2}\cdot\frac{263-1}{2}}\\ &= \left(\frac{2}{7}\right)^2\cdot(-1)\\ &= -1\end{aligned}$$

(s)

$$\begin{aligned}\left(\frac{-38}{997}\right) &= \left(\frac{-1}{997}\right)\cdot\left(\frac{38}{997}\right)\\ &= (-1)^{\frac{997-1}{2}}\cdot\left(\frac{2}{997}\right)\cdot\left(\frac{19}{997}\right)\\ &= 1\cdot(-1)^{\frac{997^2-1}{8}}\cdot\left(\frac{9}{19}\right)\cdot(-1)^{\frac{19-1}{2}\cdot\frac{997-1}{2}}\\ &= (-1)\cdot\left(\frac{3}{19}\right)^2\cdot 1\\ &= -1\end{aligned}$$

(t)

$$\begin{aligned}\left(\frac{220}{353}\right) &= \left(\frac{2}{353}\right)^2 \cdot \left(\frac{5}{353}\right) \cdot \left(\frac{11}{353}\right) \\ &= 1 \cdot \left(\frac{3}{5}\right) \cdot (-1)^{\frac{5-1}{2} \cdot \frac{353-1}{2}} \cdot \left(\frac{1}{11}\right) \cdot (-1)^{\frac{11-1}{2} \cdot \frac{353-1}{2}} \\ &= \left(\frac{2}{3}\right) \cdot (-1)^{\frac{3-1}{2} \cdot \frac{5-1}{2}} \cdot 1 \cdot 1 \cdot 1 \\ &= (-1)^{\frac{3-1}{2}} \cdot 1 \\ &= -1\end{aligned}$$